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BOOK REVIEW

Sequential Machines and Automata Theory. By T. L. Booth.

There are at least three reasons for studying automata theory: (1) it is an interesting branch of mathematics; (2) it partially attempts to answer the philosophical question, "What can a machine do?"; and (3) it is basic knowledge for the engineer interested in discrete systems. T. L. Booth's book *Sequential Machines and Automata Theory* is written for use in electrical engineering classes; therefore, it naturally emphasizes concepts of interest to the engineer.

The first thing to note about the book is that is definitely not suitable for use in a mathematics course or a course given by a mathematics oriented computer science department. As was previously stated, the book was not written for such use; however, the book is less suitable for other use than it might be for the same reasons that it is unsuitable as a mathematics text. The reasons are as follows: (1) There are numerous mathematical statements that are at best misleading. Individually, these errors are minor, but collectively they alienate the reader with a good mathematical background. (2) Definitions and theorems are not labeled as such, nor are they numbered to allow later sections to make reference to them. The statement of a definition, theorem or algorithm may be spread out through several paragraphs and it is difficult to determine the precise definition of some concepts. (3) Results that are primarily of mathematical interest are not emphasized. Examples are the characterization of regular sets by congruences of finite index, regular sets as a Boolean algebra, the relationship between homomorphisms and equivalence of machines, and the effectiveness of constructions such as the minimalization of an automaton. (This last topic is, of course, of practical importance, also.)

The book must now be discussed in terms of its usefulness to the engineering oriented student. The importance of the book lies in its extensive coverage of a wide variety of areas of automata theory. Of particular importance are the chapters on decomposition of machines, identification and control of machines, linear machines and probabilistic machines. In fact, the book will probably be most useful as a text for a second course emphasizing those topics.

At this point it should be stated that the author is at all times clear and precise in his use of English (even if the mathematical ideas expressed are a bit muddled at times). Sentences and paragraphs are written in such a manner that they do not have to be reread several times to be understood.

After an interesting and motivating introductory chapter, the second chapter presents briefly some of the basic mathematics needed to read the book. The following is a sample of some of the mathematical idiosyncrasies in chapter 2: (1) $\vee$ and $\wedge$ are used for set union and intersection. (2) The cross product of sets is properly defined as a collection of ordered pairs, but the definitions of relation and function are vague. In reference to Boolean functions it is stated that "Two functions are equal if and only if they define the same mapping." (3) It is claimed that $(R \times S) \times T = R \times (S \times T)$. (4) No attempt is made to show that lattices, Boolean algebras, semi-groups, groups, rings, and even automata, if properly defined, are all examples of the same kind of mathematical system. In fact, lattices and Boolean algebras are called "logic systems", while groups, rings, etc., are called "mathematical systems." A unified treatment of these topics should make their understanding easier for both the engineer and the mathematician. The reader should learn the material presented in chapter 2 elsewhere and use this section as a brief review and introduction to the author's notation.

Chapter 3 contains the basic definitions and properties of automata. The discussion is clear and precise and the reader is guided by many helpful diagrams. The continual switching from the Moore model to the Mealy model introduces some confusion; one model could be used to illustrate the basic principles and then the relationship between them could be shown. In order to develop the reader's intuition about the behavior of automata, more time could be spent at this point on basic concepts; for example, the relationship between mappings defined and sets accepted by automata, the construction of automata accepting the intersection and complement of sets accepted by given automata, the nature of the periodicity of sets accepted by automata and some examples of mappings not realizable or sets not accepted by automata.

The technique of minimizing the number of states in an automaton by constructing a sequence of state partitions is well presented. However, the proof that this technique works is incomplete because equivalence of machines is defined in terms of relationships between the states of the machines. Two machines should be equivalent if and only if they both effect the same computation. It must then be shown that the minimalization process given works according to that definition. This error is not repeated in the discussion of probabilistic machines.

Transition systems (or non-deterministic automata) and the subset construction are not mentioned; however, there is an excellent section about a closely related subject, the incompletely specified machine. There is an exercise of special interest which shows that the minimal incomplete machine may not be constructed by completing the given machine in all possible ways and minimizing each.

The chapter on machine decomposition properly follows the theory developed by Hartmanis and Stearns, rather than the more abstract algebraic theories. The reader is motivated at the beginning by a clear discussion of parallel, series, and feedback composition.

In this section, logical nets are discussed and an example of a net not realizable by an automaton is given. This would be a natural place to discuss briefly the precise characterization of logical nets equivalent to automata as given by Copi, Elgot and Wright (1958, 1964).

A chapter, "Measurement, Control and Identification of Sequential Machines", is one of the best of the book. Two general classes of problems are considered. In control problems, the machine is completely or partially known and the problem is to cause the machine to behave in a certain way by presenting it with the appropriate input sequence. For example, which sequence can be applied to bring a machine to a given state if its current state is unknown? The other class of problems, originally considered by Moore, concerns learning something about the structure of an unknown automaton by observing its behavior. Some of these problems are particularly interesting because the experiment may alter the condition of the machine. Consider the problem of the demolition expert trying to figure out how an unknown bomb works.

The various problems are carefully described and explicit procedures for solving them are given. The results are summarized in an easily read table.

The information-lossless machine, first investigated by Huffman, is given special attention. These machines have the property that the input sequence can be determined from the initial state and the output sequence. Any automaton used as an encoder must have this property and in this case the job of the cryptanalyst is one of solving identification problems.

The proof that regular expressions define exactly the sets accepted by automata is well presented after some more difficulties with basic concepts. "Two regular expressions describe the same regular event if and only if they describe the same subset." An attempt is made to distinguish between a regular expression and the set it defines, but the distinction is quickly ignored.

The construction of a regular expression from a given automaton involves the solution (for X) of the set equation $X = X \cdot A \cup B$. The exact nature of the solutions of this equation is given only in a problem that the reader should find fun to solve. The construction involves solving the equation only when $\lambda \notin A$ (λ is the empty string) and in this case $X = B \cdot A^*$ is a unique solution. Obtaining this unique solution is the key step in a simple and elegant method of obtaining the desired regular expression.

There are at least two approaches to the problem of constructing an automaton from a regular expression. One uses transition systems (non-deterministic automata) and the other uses derivatives of regular expressions. The latter approach is used in the text and although the proof that it works is probably as complicated as the other method, the result is an algorithm complicated somewhat only by the requirement that the process must be terminated by determining when all different derivatives of an expression have been found.

The chapter on linear machines is one of the most important in the book. It is preceded by a chapter on linear algebra. The author seems much more at home with these topics than the ones involving "abstract" algebra and the reader should learn a great deal about mathematics and machines by studying these chapters. One has the feeling that machines are being discussed and yet mathematical tools are used whenever necessary or helpful. If all chapters were of the same quality as these two, the book would be outstanding.

There is one chapter devoted to Turing machines, recursive functions and decision problems and one chapter devoted to artificial languages. Except for the finite state languages, these topics do not have much to do with the rest of the book, as it is written. However, the introduction to these topics provided in these chapters is certainly better than no exposure at all.

The term "function" is defined again, this time as a "definite correspondence", the important requirement that an encoding of strings to integers be one-to-one and effective is overlooked, the fundamental theorem about context-free sets (page 413) is stated as a theorem about finite state languages and there is a very misleading example in each chapter. The functions $C_n^k(x_1, x_2, \cdots, x_n) = k$ are shown to be recursive by applying the recursion schema to the *superscript*. Each function is recursive simply because it is the composition of the successor and zero functions. An example of a context-sensitive grammar includes the production $x\sigma \rightarrow xx$, where x is a *variable*, allowing the deduction $abb\sigma \rightarrow abbabb$. Such rules are not admissible in phrase-structure grammars.

The final chapters deal with discrete random processes and their occurrence in automata. The use of mathematics is much more extensive in this part of the book and at times it is difficult to recall that some type of machines is the object of study. The reader is not given the chance to build up his intuition by examining, for example, the simple concepts presented by Rabin (1963, 1964). The coverage of random processes is extensive, including a discussion of machines subjected to random input, as well as machines whose internal behavior is probabilistic.

In each section there is an excellent set of exercises and at the end of each chapter there is an excellent set of "home problems". The separation of "exercises" from "problems" is recommended to any writer.

In summary, the following is suggested: the student of automata theory should have a basic knowledge of abstract and linear algebra; he should learn the fundamentals of automata theory from some other source (Harrison's book (1965) is excellent); then Booth's chapters on decomposition, identification and control, linear machines and probabilistic machines will provide a wealth of information for further study.

Finally, it is recommended that any future writers of automata theory texts carefully examine the excellent mathematical approach of Büchi (1966). In this presentation, automata and mathematical systems of the usual kind and many standard algebraic techniques may be applied. (For example, homomorphisms of automata provide the key to the minimalization problem.) The theory may be readily generalized to tree automata and automata in category theory. The writer should use all aspects of this approach which provide a more convenient presentation or a better understanding for the reader, whatever his background.

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REFERENCES

BÜCHI, J. R. (1966). Algebraic theory of feedback in discrete systems, Part I *in* "Automata Theory," Academic Press, Inc., New York.

COPI, I. M., ELGOT, C. C., AND WRIGHT, J. B. (1958, 1964), Realizations of events by logical nets, *Journal Assoc. Computing Machinery*, 5, 181–196; also *in* "Sequential Machines" (E. F. Moore, Ed.), Addison Wesley, Reading, Massachusetts.

HARRISON, M. A. (1965). "Introduction to Switching and Automata Theory." McGraw-Hill, New York.

RABIN, M. O. (1963, 1964). Probabilistic automata, *Inform. Control* 6, 230–245; also *in* "Sequential Machines" (E. F. Moore, Ed.), Addison Wesley, Reading, Massachusetts.